Applied Mathematical Modelling 39 (2015) 1577–1586

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Process improvement capability index with cost – A modeling method of mathematical programming

Kuen-Suan Chen [a,*], Shiang-Li Yang [a], Hsi-Tien Chen [b]

[a] Department of Industrial Engineering and Management, National Chin-Yi University of Technology, Taichung 411, Taiwan
[b] Department of Leisure Industry Management, National Chin-Yi University of Technology, Taichung 411, Taiwan

ARTICLE INFO

Article history:
Received 27 March 2011
Received in revised form 6 January 2013
Accepted 10 September 2014
Available online 2 October 2014

Keywords:
Process capability index
Process improvement capability index
Supply chain
Mathematical programming model

ABSTRACT

To improve productivity and remain competitive, businesses nowadays introduce the process capability index (C_{pm}) to evaluate the quality of their products in an effort to improve them and cut down on operation costs. This is because C_{pm} can clearly reflect process loss and yield percentage (yield%) that it is widely used in the industry. When suppliers' process capability is found to be limited in terms of C_{pm}, an improvement in product quality is required and the cost of the improvement varies depending on the source of loss. Though C_{pm} is a very good index for the evaluation of process capability, it is unable to reflect suppliers' improvement costs. Thus, this paper takes a reduction in the improvement cost into consideration and proposes the process improvement capability index (C_{PIM}). The mathematical programming model is then used to assess the confidence interval of index C_{PIM} to overcome the problem of complicated estimation of index C_{PIM}. With C_{PIM}, manufacturers are able to evaluate suppliers' ability in process improvement, particularly when the suppliers' process capability is found to be limited, to effectively reduce suppliers' improvement costs, to improve the quality of products, to enhance productivity and finally to achieve the goal of sustainable operations.

1. Introduction

As far as modern businesses are concerned, quality is by no means a new concept. In their work, entitled *The Management and Control of Quality* [1], James R. Evans and William M. Lindsay related that back in 1887, William Cooper Procter, the grandson of the founder of Procter and Gamble, had already told his employees, "The first job we have is to turn out quality merchandise that consumers will buy and keep on buying. If we produce it efficiently and economically, we will earn a profit in which you will share." Procter's statement is especially crucial to managers in manufacturing and service organizations since it underscores the three keys to making substantial profits: productivity, cost, and quality, which are interrelated. Along similar lines, Deleryd [2] maintained that the main factors to being competitive are the pursuits of cost reduction, the enhancement of product quality and improvement in productivity. In this regard, many businesses nowadays bring in the process capability index and use it to assess product quality.

The process capability index itself includes product specifications, the process mean μ and the process standard deviation σ, with the three of them, the quality of products can be evaluated. When managers of quality management assess the quality of products, they should take into consideration not only the process mean μ and the process standard deviation

* Corresponding author.

http://dx.doi.org/10.1016/j.apm.2014.09.025

σ but also the specifications since the specifications vary from one product to another. Thus, businesses need to make a comparison of the process mean (μ), the process standard deviation (σ) and the product specifications so as to objectively assess the quality of products. The foregoing explains exactly what the process capability index is. To be more specific, the process capability index C_{pm} is a function value for the value of the process distribution parameter (μ and σ) and specification limits. Given what it includes, the process capability index C_{pm} can be used by managers of quality management to evaluate product quality.

Kane [3] proposed two process capability indices, C_p and C_{pk}; however, these two indices cannot reflect process loss because they are defined based on yield%. Chan et al. [4] proposed an index C_{pm}, which is able to adequately reflect process loss. Pearn et al. [5] emphasized that it is because index C_{pm} is capable of assessing process loss that it can faithfully reflect process capability, i.e., both process capability and process loss. Apart from this, Govaerts [6] also indicated that when the value of index C_{pm} is sufficiently high, it can even reflect process yield. The relation between index C_{pm} and process yield, as Govaerts [6] put it, is $Yield\% \geqq 2\Phi(3C_{pm}) - 1$. Thus, index C_{pm} can reflect not only process loss but also process yield. Index C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{1}{3\sqrt{\alpha^2 + \beta^2}} \quad (1)$$

here USL and LSL are the upper and lower specification limits, respectively; μ refers to the process mean; σ is the standard deviation of the process; T is the target value; and the equation $d = (USL - LSL)/2$. $\alpha = \sigma/d$ is the process accuracy index. With a small σ value, the process standard deviation σ is small too, which indicates that the process has considerable precision. That the process has sufficient accuracy is also indicated when $\beta = (\mu - T)/d$; that is, the precise index of the process is close to zero. The closer $\beta = (\mu - T)/d$ is to zero, the closer the process mean μ is to target value T, which implies that the expected process loss is low and the process is sufficiently accurate.

According to Phadke [7] and Pearn and Chen [8], there are two factors that result in process loss. The first is a loss from a deviation of the specification from the target, which explains a lack of process accuracy. The other is a loss caused by an enormous process variation. These two factors influence index C_{pm}. The smaller the loss caused by the two factors is, the higher the index value C_{pm} is. Conversely, the more substantial the loss resulting from the two factors is, the lower the index value C_{pm} is, which suggests that process capability is limited. Obviously, C_{pm} is an excellent index to measure the process capability.

The way to improve a limited process capability, reflected in a low value of index C_{pm}, depends on which factor causes the loss. For example, if a low value of index C_{pm} is caused by the process mean μ deviating from the target, manufacturers can simply offer suppliers' technical help. As Wei et al. [9] argue, suppliers can easily improve process capability simply by making adjustments to machines' parameters, and this at a low cost. But if the low value of index C_{pm} results from an enormous process variation, i.e. the process standard deviation σ, manufacturers have to spend much more money helping their suppliers purchase new machines and supervising the quality of raw materials because it is possibly caused by old machines and varying quality of the raw materials provided by the suppliers. Although C_{pm} is a good process capability index, it cannot reflect suppliers' improvement costs. Thus, this paper takes a reduction in the improvement cost into consideration and proposes a process improvement capability index (C_{PIM}). However, the estimate of index C_{PIM} is difficult. Based the advantages of modeling method of mathematical programming [10], this study uses the mathematical programming model to assess the confidence interval of index C_{PIM} to overcome the problem of complicated estimation of index C_{PIM}.

The remainder of this paper is organized as follows: Section 2 provides a description of the improvement capability index. Section 3 derives the estimate of $\hat{C}_{PIM}$. Section 4 discusses how the confidence intervals of index C_{PIM} are obtained by modeling method of mathematical programming. Finally, Section 5 presents the conclusions.

2. Improvement capability index

As noted above, C_{pm} does not reach the required quality standard, is a result of the lack of accuracy of the process and of a lack of precision of the process. Generally speaking, it involves a lower cost to improve a limited process capability caused by the former than by the latter. As the mentioned above, though C_{pm} can be used to evaluate process capability, it cannot reflect process improvement capability. The two situations in the following paragraphs are used as examples to explain this very assertion. We assume that there are two factories, A and B, which produce the same products. Assume that T is the target value; the equation $d = (USL - LSL)/2$; $T \pm d$ is the product's upper and lower specification limits; C_1 is a cost unit to improve process precision; and that C_2 is a cost unit to improve process accuracy. In the case of factory A, the index value C_{pm} is 0.9 under the assumption that the average deviation of the process from the target is $d/6$ and that the standard deviation of the process is $d/3$. Chen et al. [11] point out that there is need to improve process capability when the process capability index value C_{pm} is less than 1. Additionally, Linderman et al. [12] reported that with the definition of the six sigma, American Motorola Corporation allows the process deviation from target to be no more than $d/4$, that is, under 1.5σ. The deviation in factory A is within the six sigma; therefore, the main reason for the process incapability is the lack of precision of the process. Furthermore, the result of the analysis by the quality department shows that lack of precision of the process was likely brought about by a shortage of raw materials, which results in frequent change of source of raw materials in the market. Besides, given the different batches of material, the quality of the raw materials varies, which possibly led to the process incapability. It costs a great deal to solve this problem on account of the increased budget for the purchase of the materials and the strict supervision of the quality of the raw materials that was required.

In the case of factory B, the index value C_{pm} is 0.9 under the presumption that the process average deviation from the target is $d/3$ and that the process standard deviation is $d/6$. It is obvious that the process deviation is greater than $d/4$. Thus, the cause of the process incapability is the lack of accuracy of the process. And the analysis by the quality department reveals that the lack of accuracy of the process was possibly a result of defective machine parameters. Generally, the problem can be solved simply by making adjustments to the parameters of the machines.

If the cost of improving the problem that B suppliers is 1, then the cost of improving the problem that A suppliers is ten times more than that B supplier has to bear. Based on the discussion above, the process improvement capability index is defined as follows:

$$C_{PIM} = \frac{1}{3\sqrt{C_1\alpha^2 + C_2\beta^2}}. \tag{2}$$

The foregoing process improvement capability index C_{PIM} focuses on the differences between two methods of keeping costs down. For the convenience of practical application, we assume $r = C_2/C_1$, thereby simplifying the process improvement capability index to the equation below:

$$C_{PIM} = \frac{1}{3\sqrt{\alpha^2 + r\beta^2}}. \tag{3}$$

With the equation above, we determine the outcome that the values of the process improvement capability for supplier A and B are 0.31 and 0.53, respectively. It is clearly apparent that the suppliers' process capability in both cases, A and B, is quite limited since the index value C_{pm} is 0.9. Nevertheless, supplier B's process improvement capability is higher than that of A. Thus, supplier B is preferred to supplier A since supplier B is able to simply spend less money to improve process capability.

3. The estimate of $\hat{C}_{PIM}$

We assume that the characteristic of process x is normally distributed with mean μ and variance σ^2 and let $x_1, \ldots, x_n$ be a set of the random sample from normal distribution; the estimate of this is the same as that of index C_{pm}. In this section, we use the sample mean $\bar{x} = \sum_{i=1}^{n} x_i/n$ to estimate μ. Further, with the sample standard deviation, $S = \left[\sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)\right]^{1/2}$, we can calculate σ. The natural estimate of $\hat{C}_{PIM}$ is as follows:

$$\hat{C}_{PIM} = \frac{1}{3\sqrt{\hat{\alpha}^2 + r\hat{\beta}^2}}. \tag{4}$$

We presume $\hat{\alpha} = S/d$ and $\hat{\beta} = (\bar{x} - T)/d$. And then we derive the expected value, variance and mean square errors (*MSE*). ${}_2F_1\,(a,b,c,z)$ is the super geometry function. The value of the parameter is $a = \{1/2, 1\}$, $b = (j+1)/2$, $c = (j+n)/2$, $z = (n-1/n)r$. Abramowitz and Stegun argued the following:

$$E\left(\hat{C}_{PIM}\right) = C_{PIM}\sqrt{\left(1+\frac{r\beta^2}{\alpha^2}\right)\left(\frac{n-1}{2}\right)}e^{-\frac{n\beta^2}{2\alpha^2}} \times \sum_{j=0}^{\infty}\frac{\left(n\beta^2/\alpha^2 2\right)^j}{j!}\frac{\Gamma((n-1)/2+j)}{\Gamma(n/2+j)}{}_2F_1\left(\frac{1}{2}, j+\frac{1}{2}, j+\frac{n}{2}, 1-\frac{n-1}{n}r\right), \tag{5}$$

$$\begin{aligned} Var\left(\hat{C}_{PIM}\right) = C_{PIM}^2\left(1+\frac{r\beta^2}{\alpha^2}\right)\left(\frac{n-1}{2}\right) \times \Bigg\{&\sum_{j=0}^{\infty}\frac{\left(n\beta^2/\alpha^2 2\right)^j}{j!}\frac{2}{n+2j-2}{}_2F_1\left(1, j+\frac{1}{2}, j+\frac{n}{2}, 1-\frac{n-1}{n}r\right) \\ &-\left[\sum_{j=0}^{\infty}\frac{\left(n\beta^2/\alpha^2 2\right)^j}{j!}\frac{\Gamma((n-1)/2+j)}{\Gamma(n/2+j)}{}_2F_1\left(\frac{1}{2}, j+\frac{1}{2}, j+\frac{n}{2}, 1-\frac{n-1}{n}r\right)\right]^2\Bigg\}, \end{aligned} \tag{6}$$

$$MSE\ \left(\hat{C}_{PIM}\right) = Var\ \left(\hat{C}_{PIM}\right) + \left[Bias\ \left(\hat{C}_{PIM}\right)\right]^2, \tag{7}$$

$$Bias\left(\hat{C}_{PIM}\right) = C_{PIM}\left[\sqrt{\left(1+\frac{r\beta^2}{\alpha^2}\right)\left(\frac{n-1}{2}\right)}e^{-\frac{n\beta^2}{2\alpha^2}} \times \sum_{j=0}^{\infty}\frac{\left(n\beta^2/\alpha^2 2\right)^j}{j!}\frac{\Gamma((n-1)/2+j)}{\Gamma(n/2+j)}{}_2F_1\left(\frac{1}{2}, j+\frac{1}{2}, j+\frac{n}{2}, 1-\frac{n-1}{n}r\right) - 1\right]. \tag{8}$$

Obviously, the *MSE* is a function of α value and β value. $\beta = (\mu - T)/d$ is the process deviation rate. Both the high and low $|\beta|$ values indicate that the process deviates from the target T and requires adjustment. Additionally, $\alpha = \sigma/d$ reveals the extent to which the process varies; the larger the $|\alpha|$ value is, the more greatly the process varies and the more unstable the process becomes. That is, given $|\beta|$ value is closer to the target, a smaller $|\alpha|$ value implies the process is much steadier and more precise. The value of *MSE* and bias is subject to alternation due to the α and β values. The contour map below, divided into five parts under the condition that $C_{pm} = c$, illustrates how the *MSE* value is affected by the α and β values. h indicates the extent to which the process deviates and is marked 0,1,2,3,4 respectively (see Fig. 1).

The coordinates of the five parts are $\left(\alpha = \sqrt{\frac{1}{(3c)^2} - \left(\frac{h}{5^*(3c)}\right)^2},\ \beta = \frac{h}{5^*(3c)}\right)$, $h = 0, 1, 2, 3, 4$. Obviously, in the case of $h = 0$, $\mu = T$ is on the target. The larger the h value is, the more the process deviates from the target. However, while the index value is stable, the variance value is small. Practically speaking, the cases where $h = 4$ and $h = 5$ hardly occur since the index value varies little. Particularly in the case of $h = 5$, the standard deviation of the process is zero, which is pragmatically impossible. Thus, this study only focuses on the *MSE* value in the respective cases where $h = 0, 1, 2$ and 3. It is noted that the *MSE* value of $\hat{C}_{PIM}$ is subject to change due to a variety of r values. Therefore, we take into consideration not only $h = 0, 1, 2$ and 3 but also $r = 0.2, 0.4, 0.6, 0.8$ and 1.0, which are the different rates of the improvement costs. C_{PIM} is usually applied under the condition where there is process incapability and a need for an evaluation of the ability to improve the process. In other words, that the index value C_{pm} is no more than 1 is indicative that process capability is limited and that investments need to be made for an improvement. Thus, Table 1 illustrates the α, β and the *MSE* of the C_{pm}, corresponding to the r in the case of the process incapability, when the $\hat{C}_{PIM}$ is from 0.6 to 0.9. Under the condition that $h = 1$, the analysis of the interrelation among n, numbers of the sample, r, the values of the improvement costs, the *bias* and the *MSE* values are illustrated in Figs. 2–5.

4. Confidence intervals – a modeling method of mathematical programming

Since manufacturers are unable to evaluate original suppliers' process improvement capability with index C_{pm}. Thus, this section puts much emphasis on this problem, proposing the process improvement capability index C_{PIM}. Manufacturers can use index C_{PIM} to appraise suppliers' process improvement capability. Cheng [13] pointed out that if index C_{PIM} is the measurement of point estimation, the process parameters of production are generally unknown, and the average value of samples is all that can be obtained. Due to errors of samples, it is unreliable to use the estimated value to evaluate process capability and process capability improvement. Thus, this section discusses how the confidence intervals of index C_{PIM} can be used to measure suppliers' process improvement capability. However, it is not easy to figure out the confidence intervals because the estimate of index C_{PIM} distribution is very complex. This will result in difficulties in the practical application of index C_{PIM}.

C_{PIM} is a function of the process parameters α and β. Besides, the process parameters α and β can be obtained by multiplying μ and σ respectively by a constant. The match of (α, β) represents a process. Thus, α and β of $100(1-\gamma)\%$ joint confidence intervals, as a matter of fact, can be easily obtained. According to the Boole inequality, how to figure out α and β joint confidence intervals is demonstrated in the following formulas:

$$\alpha = [\alpha_L, \alpha_U] = \left[\sqrt{\frac{(n-1)\times c_4^2 \times \hat{\alpha}^2}{\chi^2_{1-\gamma/4}(n-1)}}, \sqrt{\frac{(n-1)\times c_4^2 \times \hat{\alpha}^2}{\chi^2_{\gamma/4}(n-1)}}\right], \tag{9}$$

$$\beta = [\beta_L, \beta_U] = \left[\left(\hat{\beta} - t_{\gamma/4}(n-1)\times c_4 \times \frac{\hat{\alpha}}{\sqrt{n}}\right), \left(\hat{\beta} + t_{r/4}(n-1)\times c_4 \times \frac{\hat{\alpha}}{\sqrt{n}}\right)\right], \tag{10}$$

where $c_4 = \sqrt{2/(n-1)}\Gamma[n/2]/\Gamma[(n-1)/2]$; $t_{r/4}\ (n-1)$ is the t-distribution with $(n-1)$ degrees of freedom. The right tail's dimension of $t_{r/4}\ (n-1)$ is $(r/4)$. If $\chi^2_{1-\gamma/4}(n-1)$ is a chi-square distribution with $(n-1)$ degrees of freedom, then the right tail's dimension of $\chi^2_{1-\gamma/4}(n-1)$ is $(1-\gamma/4)$. If $\chi^2_{\gamma/4}(n-1)$ is a chi-square distribution with (n-1) degrees of freedom, then the right tail's dimension of $\chi^2_{\gamma/4}(n-1)$ is $(r/4)$. The joint confidence intervals of α and β are able to reflect the various types of the process.

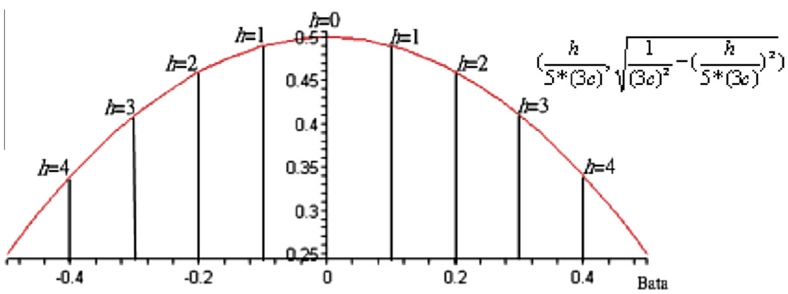

Fig. 1. The contour map ($C_{pm} = c$).

Table 1
The b*ias* and *MSE* values for various C_{PIM}.

C	r	n	$H = 0$		$H = 1$		$H = 2$		$H = 3$	
			C_{PIM} = 0.90000		C_{PIM} = 0.91476		C_{PIM} = 0.96379		C_{PIM} = 1.06660	
			Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
C_{pm} = 0.9, 0.8										
0.9	0.2	10	.07123	.07043	.07120	.07116	.07077	.07316	.06843	.07535
		30	.02077	.01605	.02080	.01632	.02080	.01714	.02038	.01845
		50	.01215	.00898	.01217	.00914	.01219	.00963	.01197	.01045
		70	.00858	.00622	.00860	.00634	.00862	.00669	.00847	.00729
		90	.00664	.00476	.00665	.00485	.00666	.00513	.00655	.00559
	0.4		C_{PIM} = 0.90000		C_{PIM} = 0.91100		C_{PIM} = 0.94658		C_{PIM} = 1.01645	
		10	.05938	.06506	.05878	.06492	.05657	.06404	.05109	.06031
		30	.01750	.01566	.01734	.01569	.01674	.01571	.01523	.01530
		50	.01026	.00884	.01016	.00887	.00981	.00891	.00895	.00873
		70	.00725	.00616	.00719	.00618	.00694	.00621	.00633	.00611
		90	.00561	.00472	.00556	.00474	.00537	.00477	.00490	.00470
	0.6		C_{PIM} = 0.90000		C_{PIM} = 0.90729		C_{PIM} = 0.93026		C_{PIM} = 0.97276	
		10	.04872	.06129	.04828	.06093	.04658	.05926	.04204	.05385
		30	.01434	.01535	.01422	.01528	.01374	.01495	.01249	.01387
		50	.00840	.00874	.00833	.00870	.00805	.00853	.00733	.00795
		70	.00594	.00611	.00589	.00608	.00569	.00596	.00519	.00557
		90	.00459	.00469	.00455	.00467	.00440	.00458	.00401	.00428
	0.8		C_{PIM} = 0.90000		C_{PIM} = 0.90362		C_{PIM} = 0.91476		C_{PIM} = 0.93426	
		10	.03900	.05856	.03921	.05827	.03932	.05664	.03722	.05076
		30	.01128	.01511	.01138	.01503	.01153	.01462	.01108	.01324
		50	.00658	.00866	.00664	.00861	.00675	.00837	.00651	.00760
		70	.00464	.00606	.00469	.00603	.00477	.00587	.00461	.00533
		90	.00358	.00467	.00362	.00464	.00369	.00452	.00357	.00411
	1.0		C_{PIM} = 0.90000		C_{PIM} = 0.90000		C_{PIM} = 0.90000		C_{PIM} = 0.90000	
		10	.03001	.05657	.03123	.05647	.03396	.05522	.03475	.04923
		30	.00830	.01493	.00880	.01490	.00993	.01456	.01046	.01300
		50	.00478	.00859	.00510	.00858	.00581	.00838	.00616	.00748
		70	.00336	.00603	.00359	.00602	.00410	.00588	.00436	.00525
		90	.00259	.00465	.00277	.00464	.00317	.00453	.00338	.00405
0.8	0.2		C_{PIM} = 0.80000		C_{PIM} = 0.81312		C_{PIM} = 0.85671		C_{PIM} = 0.94809	
		10	.06332	.05565	.06329	.05622	.06291	.05781	.06083	.05954
		30	.01846	.01268	.01849	.01290	.01849	.01354	.01811	.01458
		50	.01080	.00709	.01082	.00722	.01083	.00761	.01064	.00826
		70	.00763	.00492	.00764	.00501	.00766	.00529	.00753	.00576
		90	.00590	.00376	.00591	.00383	.00592	.00405	.00583	.00442
	0.4		C_{PIM} = 0.80000		C_{PIM} = 0.80978		C_{PIM} = 0.84141		C_{PIM} = 0.90351	
		10	.05278	.05140	.05225	.05130	.05028	.05060	.04541	.04765
		30	.01556	.01237	.01541	.01240	.01488	.01241	.01354	.01209
		50	.00912	.00699	.00903	.00701	.00872	.00704	.00795	.00690
		70	.00645	.00487	.00639	.00488	.00617	.00491	.00563	.00483
		90	.00499	.00373	.00494	.00375	.00477	.00377	.00436	.00371
	0.6		C_{PIM} = 0.80000		C_{PIM} = 0.80648		C_{PIM} = 0.82690		C_{PIM} = 0.86468	
		10	.04331	.04842	.04292	.04814	.04140	.04682	.03737	.04255
		30	.01275	.01213	.01264	.01207	.01221	.01181	.01110	.01096
		50	.00747	.00690	.00740	.00688	.00715	.00674	.00652	.00628
		70	.00528	.00482	.00523	.00480	.00506	.00471	.00461	.00440
		90	.00408	.00371	.00405	.00369	.00391	.00362	.00357	.00338
	0.8		C_{PIM} = 0.80000		C_{PIM} = 0.80322		C_{PIM} = 0.81312		C_{PIM} = 0.83045	
		10	.03466	.04627	.03485	.04604	.03495	.04475	.03308	.04010
		30	.01002	.01194	.01012	.01187	.01025	.01155	.00985	.01046
		50	.00585	.00684	.00591	.00680	.00600	.00662	.00579	.00601
		70	.00412	.00479	.00417	.00477	.00424	.00464	.00410	.00421
		90	.00319	.00369	.00322	.00367	.00328	.00357	.00317	.00324
	1.0		C_{PIM} = 0.80000		C_{PIM} = 0.80000		C_{PIM} = 0.80000		C_{PIM} = 0.80000	
		10	.02668	.04470	.02776	.04462	.03018	.04363	.03089	.03890
		30	.00737	.01180	.00782	.01178	.00882	.01150	.00930	.01027
		50	.00425	.00679	.00453	.00678	.00516	.00662	.00547	.00591
		70	.00299	.00477	.00319	.00476	.00365	.00465	.00388	.00415
		90	.00230	.00367	.00246	.00367	.00282	.00358	.00300	.00320
C_{pm} = 0.7, 0.6										
0.7	0.2		C_{PIM} = 0.70000		C_{PIM} = 0.71148		C_{PIM} = 0.74962		C_{PIM} = 0.82958	
		10	.05540	.04261	.05538	.04305	.05505	.04426	.05322	.04558
		30	.01615	.00971	.01617	.00987	.01618	.01037	.01585	.01116
		50	.00945	.00543	.00946	.00553	.00948	.00583	.00931	.00632

(*continued on next page*)

Table 1 (*continued*)

C	r	n	$H = 0$		$H = 1$		$H = 2$		$H = 3$	
			C_{PIM} = 0.90000		C_{PIM} = 0.91476		C_{PIM} = 0.96379		C_{PIM} = 1.06660	
			Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
		70	.00668	.00376	.00669	.00383	.00670	.00405	.00659	.00441
		90	.00516	.00288	.00517	.00293	.00518	.00310	.00510	.00338
	0.4		C_{PIM} = 0.70000		C_{PIM} = 0.70855		C_{PIM} = 0.73623		C_{PIM} = 0.79057	
		10	.04618	.03936	.04571	.03927	.04400	.03874	.03974	.03649
		30	.01361	.00947	.01349	.00949	.01302	.00950	.01185	.00925
		50	.00798	.00535	.00790	.00537	.00763	.00539	.00696	.00528
		70	.00564	.00373	.00559	.00374	.00540	.00376	.00493	.00369
		90	.00436	.00286	.00432	.00287	.00418	.00289	.00381	.00284
	0.6		C_{PIM} = 0.70000		C_{PIM} = 0.70567		C_{PIM} = 0.72354		C_{PIM} = 0.75659	
		10	.03790	.03707	.03755	.03686	.03623	.03585	.03270	.03257
		30	.01116	.00929	.01106	.00924	.01068	.00904	.00971	.00839
		50	.00653	.00529	.00648	.00526	.00626	.00516	.00570	.00481
		70	.00462	.00369	.00458	.00368	.00443	.00361	.00403	.00337
		90	.00357	.00284	.00354	.00283	.00342	.00277	.00312	.00259
	0.8		C_{PIM} = 0.70000		C_{PIM} = 0.70282		C_{PIM} = 0.71148		C_{PIM} = 0.72665	
		10	.03033	.03543	.03050	.03525	.03058	.03426	.02895	.03070
		30	.00877	.00914	.00885	.00909	.00897	.00884	.00862	.00801
		50	.00511	.00524	.00517	.00521	.00525	.00507	.00506	.00460
		70	.00361	.00367	.00365	.00365	.00371	.00355	.00358	.00323
		90	.00279	.00282	.00282	.00281	.00287	.00273	.00277	.00248
	1.0		C_{PIM} = 0.70000		C_{PIM} = 0.70000		C_{PIM} = 0.70000		C_{PIM} = 0.70000	
		10	.02334	.03422	.02429	.03416	.02641	.03341	.02703	.02978
		30	.00645	.00903	.00684	.00902	.00772	.00881	.00813	.00787
		50	.00372	.00520	.00397	.00519	.00452	.00507	.00479	.00453
		70	.00261	.00365	.00279	.00364	.00319	.00356	.00339	.00318
		90	.00201	.00281	.00215	.00281	.00247	.00274	.00263	.00245
0.6	0.2		C_{PIM} = 0.60000		C_{PIM} = 0.60984		C_{PIM} = 0.64253		C_{PIM} = 0.71107	
		10	.04749	.03130	.04746	.03162	.04718	.03252	.04562	.03349
		30	.01384	.00713	.01386	.00725	.01387	.00762	.01359	.00820
		50	.00810	.00399	.00811	.00406	.00812	.00428	.00798	.00465
		70	.00572	.00277	.00573	.00282	.00574	.00297	.00565	.00324
		90	.00442	.00212	.00443	.00216	.00444	.00228	.00437	.00248
	0.4		C_{PIM} = 0.60000		C_{PIM} = 0.60733		C_{PIM} = 0.63105		C_{PIM} = 0.67763	
		10	.03958	.02891	.03918	.02885	.03771	.02846	.03406	.02681
		30	.01167	.00696	.01156	.00697	.01116	.00698	.01015	.00680
		50	.00684	.00393	.00677	.00394	.00654	.00396	.00596	.00388
		70	.00483	.00274	.00479	.00275	.00463	.00276	.00422	.00271
		90	.00374	.00210	.00371	.00211	.00358	.00212	.00327	.00209
	0.6		C_{PIM} = 0.60000		C_{PIM} = 0.60486		C_{PIM} = 0.62017		C_{PIM} = 0.64851	
		10	.03248	.02724	.03219	.02708	.03105	.02634	.02803	.02393
		30	.00956	.00682	.00948	.00679	.00916	.00664	.00833	.00617
		50	.00560	.00388	.00555	.00387	.00537	.00379	.00489	.00353
		70	.00396	.00271	.00392	.00270	.00379	.00265	.00346	.00247
		90	.00306	.00209	.00303	.00208	.00293	.00204	.00268	.00190
	0.8		C_{PIM} = 0.60000		C_{PIM} = 0.60241		C_{PIM} = 0.60984		C_{PIM} = 0.62284	
		10	.02600	.02603	.02614	.02590	.02621	.02517	.02481	.02256
		30	.00752	.00672	.00759	.00668	.00769	.00650	.00739	.00589
		50	.00438	.00385	.00443	.00383	.00450	.00372	.00434	.00338
		70	.00309	.00270	.00313	.00268	.00318	.00261	.00307	.00237
		90	.00239	.00207	.00242	.00206	.00246	.00201	.00238	.00183
	1.0		C_{PIM} = 0.60000		C_{PIM} = 0.60000		C_{PIM} = 0.60000		C_{PIM} = 0.60000	
		10	.02001	.02514	.02082	.02510	.02264	.02454	.02317	.02188
		30	.00553	.00664	.00587	.00662	.00662	.00647	.00697	.00578
		50	.00319	.00382	.00340	.00381	.00387	.00372	.00410	.00333
		70	.00224	.00268	.00239	.00268	.00274	.00261	.00291	.00233
		90	.00172	.00207	.00185	.00206	.00212	.00201	.00225	.00180

Clearly, there is a strong possibility that the actual processes α and β fall in a joint confidence region $J = [\alpha_L, \alpha_U] \times [\beta_L, \beta_U]$. In other words, the actual process can be regarded as being located in the joint confidence region. Because the estimate of index C_{PIM} distribution is complicated, this study proposes a modeling method of mathematical programming to solve the values of (α, β) and the confidence interval of C_{PIM}. Thus, $C_{PIM} = 1/3\sqrt{\alpha^2 + r\beta^2}$ is set to as an objective function of mathematical programming model and joint confidence region J is a constraint area of (α, β). It is easy to determine the maximum and minimum of C_{PIM}, i.e. $100(1-\gamma)\%$ upper and lower confidence limits. The general formulas are as follows:

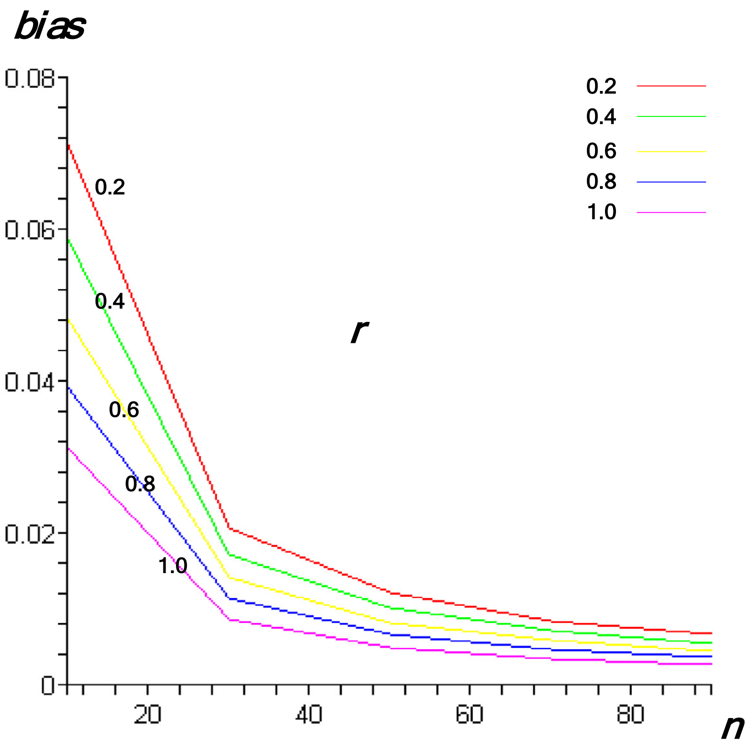

Fig. 2. The relationship between *n* and *bias* for various *r*.

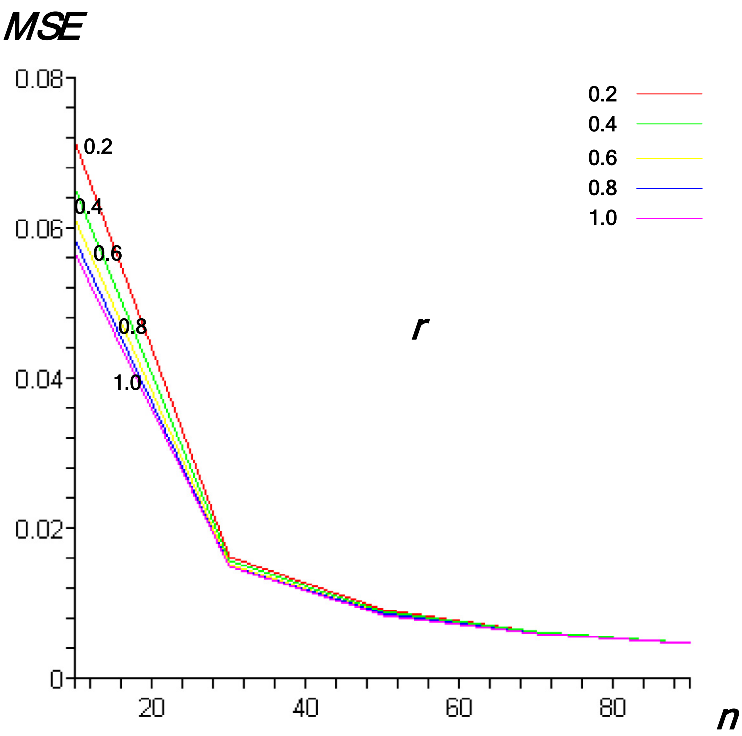

Fig. 3. The relationship between *n* and *MSE* for various *r*.

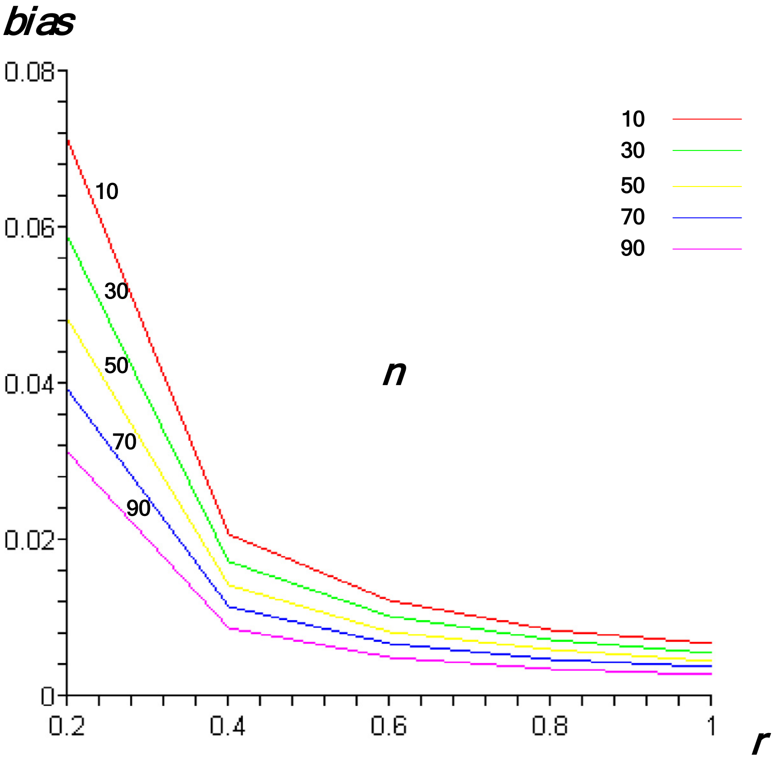

Fig. 4. The relationship between r and *bias* for various n.

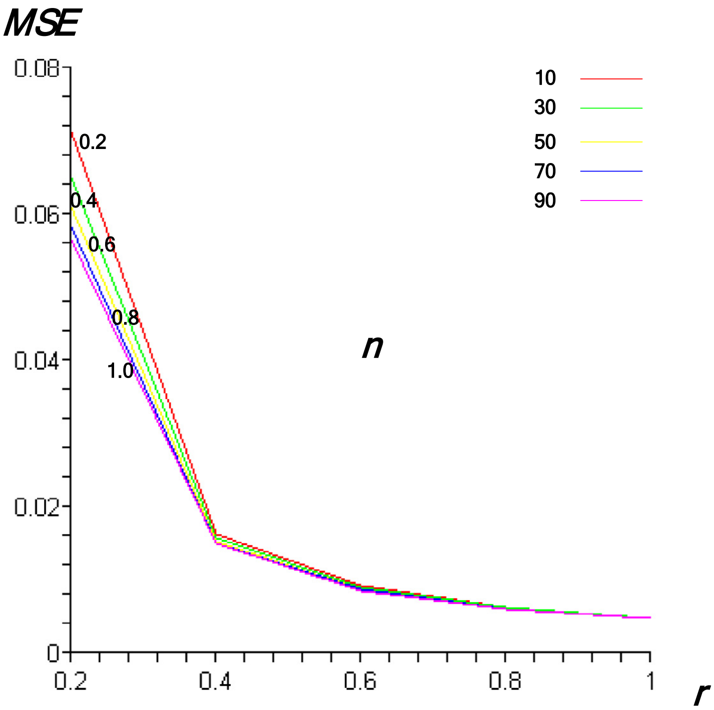

Fig. 5. The relationship between r and *MSE* for various n.

$$\begin{cases} \text{Max} & C_{PIM} \\ \text{s.t..} & \alpha_L \leqslant \alpha \leqslant \alpha_U \\ & \beta_L \leqslant \beta \leqslant \beta_U \end{cases}, \tag{11}$$

$$\begin{cases} \text{Max} & C_{PIM} \\ \text{s.t.} & \alpha_L \leqslant \alpha \leqslant \alpha_U \\ & \beta_L \leqslant \beta \leqslant \beta_U \end{cases}. \tag{12}$$

The probability of the confidence region for α and β is $100(1-\gamma)\%$, therefore, we can obtain the $100(1-\gamma)\%$ confidence intervals of C_{PIM}. For instance, the goods a supplier has number 100. We presume that the characteristics of product quality are as follows: mean = 0.16, standard deviation = 0.31, improvement rate = 0.8 and confidence level = 0.95 (i.e. $\gamma = 0.05$). According to Vännman [14], the upper confidence limit value and the lower confidence limit value of α and β can be obtained as follows:

$$\alpha_L = \sqrt{\frac{(n-1)\times c_4^2 \times \hat{\alpha}^2}{\chi^2_{1-r/4}(n-1)}} = 0.266560, \tag{13}$$

$$\alpha_U = \sqrt{\frac{(n-1)\times c_4^2 \times \hat{\alpha}^2}{\chi^2_{r/4}(n-1)}} = 0.367280, \tag{14}$$

$$\beta_L = \hat{\beta} - t_{r/4}(n-1)\times c_4 \times \frac{\hat{\alpha}}{\sqrt{n}} = 0.159514, \tag{15}$$

$$\beta_U = \hat{\beta} + t_{r/4}(n-1)\times c_4 \times \frac{\hat{\alpha}}{\sqrt{n}} = 0.160486, \tag{16}$$

where $c_4 = \sqrt{2/(n-1)}\Gamma[n/2]/\Gamma[(n-1)/2] = 0.997478$.

The maximum and minimum of α and β are obtained as follows:

$$\begin{cases} \text{Max} & C_{PIM} \\ \text{s.t.} & 0.266560 \leqslant \alpha \leqslant 0.367280 \\ & 0.159514 \leqslant \beta \leqslant 0.160486 \end{cases}, \tag{17}$$

$$\begin{cases} \text{Max} & C_{PIM} \\ \text{s.t.} & 0.266560 \leqslant \alpha \leqslant 0.367280 \\ & 0.159514 \leqslant \beta \leqslant 0.160486 \end{cases}. \tag{18}$$

The joint upper confidence limit value and lower confidence limit value of α and β are $\alpha = [0.266560, 0.367280]$ and $\beta = [0.159514, 0.160486]$, respectively. With the joint confidence intervals of α and β and using the mathematical formula of index C_{PIM} , then we can obtain the maximum and minimum of C_{PIM}, that is, the upper confidence limit and lower confidence limit of index C_{PIM}, can be obtained. We solve that Max C_{PIM} = 1.100885 and Min C_{PIM} = 0.845984 respectively, i.e., the upper confidence limit of index C_{PIM} is equal to 1.100885, while the lower confidence limit of index C_{PIM} is equal to 0.845984, which suggests that the supplier's process improvement capability index C_{PIM} value is between 0.8 and 1.1 and that the confidence interval of C_{PIM} is [0.85, 1.10].

5. Conclusions

In that the pursuit of cost reduction, the enhancement of product quality and the improvement of productivity are key to maintaining competitiveness, many modern businesses nowadays introduce the process capability index and use it as a management tool for measuring product quality. Particularly in the manufacturing industry, manufacturers have to know how to select and evaluate suppliers and subcontractors, how to enhance the quality of components, how to face up to the challenges brought about by both global operations and the increase in Build-to-Order (BTO), and finally how to quickly transform domestic businesses into transnational ones. To be sure, these have all become great concerns of manufacturers.

To evaluate product quality requires product specifications, the process mean μ and the process standard deviation σ. According to Phadke [7] and Pearn and Chen [8], there are two factors in process loss. One is a loss that results from the deviation of the specification from the target, which reveals a lack of accuracy in the process. The other is a loss caused by enormous variation in the process. Index C_{pm} is subject to alternation due to the amount of loss brought about by these two factors. The smaller the loss caused by the two factors is, the higher the index value C_{pm} is. Conversely, the more substantial the loss resulting from the two factors is, the lower the index value C_{pm} is, which suggests that process capability is limited. Obviously, C_{pm} is an excellent index to measure process capability. However, When suppliers' process capability is found to be C_{pm} incapability, i.e., a low index value C_{pm}, an improvement in product quality is required. The cost of the improvement varies depending on the source of loss.

As mentioned above, if the low value of index C_{pm} is likely brought about by the varying quality of raw materials and old machines, manufacturers have to spend much more money to help their suppliers to purchase new machines and to oversee the quality of the raw materials supplied by their original suppliers. This study particularly takes the reduction in the cost of improvement into consideration and thus proposes index C_{PIM} so as to evaluate the suppliers' process improvement capability. However, because the estimate of index C_{PIM} distribution is very complex, this study proposes a modeling method of mathematical programming to solve this problem.

Thus, manufacturers can use index C_{pm} to evaluate the suppliers' process capability in the first place. If the suppliers' process capability is proven to be limited in the evaluation, the manufacturers should find out which one, the lack of accuracy in the process or the lack of precision in the process, leads to the process incapability, and then use index C_{PIM} to measure suppliers' ability to improve the process. With these two indices, manufacturers are able to select the most capable suppliers with the greatest potential to assist long-term suppliers enhance product quality, to cut down on the cost of improvements, to improve productivity, to achieve a win–win relationship between manufacturers and suppliers and to, therefore, make the supply chain considerably more competitive.

References

[1] R.E. James, M.L. William, The Management and Control of Quality, 5E. By South-Western, a division of Thomson Learning Inc, 2002.
[2] M. Deleryd, A pragmatic view on process capability studies, Int. J. Prod. Econ. 58 (1999) 319–330.
[3] V.E. Kane, Process capability indices, J. Qual. Technol. 18 (1986) 41–52.
[4] L.K. Chan, S.W. Cheng, F.A. Spiring, A new measure of process capability: C_{pm}, J. Qual. Technol. 20 (1988) 162–175.
[5] W.L. Pearn, S. Kotz, N.L. Johnson, Distributional and inferential properties of process capability indices, J. Qual. Technol. 24 (1992) 216–231.
[6] B. Govaerts, Private Communication to S. Kotz, 1994.
[7] M.S. Phadke, Quality Engineering Using Robust Design, AT&T Bell Laboratories, New Jersey, 1989.
[8] W.L. Pearn, K.S. Chen, Process improvement capability analysis based on expected process loss, J. Chin. Stat. Assoc. 35 (1997) 151–160.
[9] P.S. Wei, C.W. Wen, Missed joint induced by thermoelectric magnetic field in electron-beam welding dissimilar metals-experiment and scale analysis, Metall. Mater. Trans. B 33 (2002) 765–773.
[10] A. Kokangul, Z. Susuz, Integrated analytical hierarch process and mathematical programming to supplier selection problem with quantity discount, Appl. Math. Modell. 33 (3) (2009) 1417–1429.
[11] K.S. Chen, M.L. Huang, R.K. Li, Process capability analysis for an entire product, Int. J. Prod. Res. 39 (2001) 4077–4087.
[12] K. Linderman, Roger G. Schroeder, Srilata Zaheer, Adrian S. Choo, Six sigma: a goal-theoretic perspective, J. Oper. Manage. 21 (2003) 193–203.
[13] S.W. Cheng, Practical implementation of the process capability indices, Qual. Eng. 7 (1994–1995) 239–259.
[14] K. Vännman, A unified approach to capability indices, Stat. Sin. 5 (1995) 805–820.